BOARD QUESTION PAPER : MARCH 2022
MATHEMATICS AND STATISTICS

Time: 3 Hrs. **Max. Marks: 80**

General instructions:

*The question paper is divided into **FOUR** sections.*

(1) **Section A:** Q.1 contains **Eight** multiple choice type of questions, each carrying **Two** marks.
Q.2 contains **Four** very short answer type questions, each carrying **one** mark.

(2) **Section B:** Q.3 to Q. 14 contain **Twelve** short answer type questions, each carrying **Two** marks. (Attempt any **Eight**)

(3) **Section C:** Q.15 to Q. 26 contain **Twelve** short answer type questions, each carrying **Three** marks. (Attempt any **Eight**)

(4) **Section D:** Q.27 to Q. 34 contain **Eight** long answer type questions, each carrying **Four** marks. (Attempt any **Five**)

(5) Use of log table is allowed. Use of calculator is not allowed.

(6) Figures to the right indicate full marks.

(7) Use of graph paper is not necessary. Only rough sketch of graph is expected.

(8) For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabet, e.g. (a)……. / (b)…….. /(c)…….. /(d)…….., etc. No marks shall be given, if ONLY the correct answer or the alphabet of correct answer is written. Only the first attempt will be considered for evaluation.

(9) Start answer to each section on a new page.

SECTION – A

Q.1. Select and write the correct answer for the following multiple choice type of questions: **[16]**

(i) The negation of $p \wedge (q \rightarrow r)$ is _______.

(a) $\sim p \wedge (\sim q \rightarrow \sim r)$ (b) $p \vee (\sim q \vee r)$
(c) $\sim p \wedge (\sim q \rightarrow r)$ (d) $p \rightarrow (q \wedge \sim r)$ (2)

(ii) In ΔABC if $c^2 + a^2 - b^2 = ac$, then $\angle B =$ _______.

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$ (2)

(iii) Equation of line passing through the points (0, 0, 0) and (2, 1, –3) is _______.

(a) $\frac{x}{2} = \frac{y}{1} = \frac{z}{-3}$ (b) $\frac{x}{2} = \frac{y}{-1} = \frac{z}{-3}$
(c) $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ (d) $\frac{x}{3} = \frac{y}{1} = \frac{z}{2}$ (2)

(iv) The value of $\hat{i}\cdot(\hat{j}\times\hat{k}) + \hat{j}\cdot(\hat{k}\times\hat{i}) + \hat{k}\cdot(\hat{i}\times\hat{j})$ is _______.

(a) 0 (b) –1 (c) 1 (d) 3 (2)

(v) If $f(x) = x^5 + 2x - 3$, then $(f^{-1})'(-3) =$ _______.

(a) 0 (b) –3 (c) $-\frac{1}{3}$ (d) $\frac{1}{2}$ (2)

(vi) The maximum value of the function $f(x) = \frac{\log x}{x}$ is _______.

(a) e (b) $\frac{1}{e}$ (c) e^2 (d) $\frac{1}{e^2}$ (2)

(vii) If $\int \frac{dx}{4x^2 - 1} = A \log\left(\frac{2x-1}{2x+1}\right) + c$, then A = _______.

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$ (2)

(viii) If the p.m.f of a r.v.X is

$P(x) = \frac{c}{x^3}$, for $x = 1, 2, 3$

$= 0$, otherwise,

then $E(X) =$ ________

(a) $\frac{216}{251}$ (b) $\frac{294}{251}$ (c) $\frac{297}{294}$ (d) $\frac{294}{297}$ (2)

Q.2. Answer the following questions: **[4]**

(i) Find the principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ (1)

(ii) Write the separate equations of lines represented by the equation $5x^2 - 9y^2 = 0$ (1)

(iii) If $f'(x) = x^{-1}$, then find $f(x)$ (1)

(iv) Write the degree of the differential equation

$(y''')^2 + 3(y'') + 3xy' + 5y = 0$ (1)

SECTION – B

Attempt any EIGHT of the following questions: **[16]**

Q.3. Using truth table verify that:

$(p \wedge q) \vee \sim q \equiv p \vee \sim q$ (2)

Q.4. Find the cofactors of the elements of the matrix $\begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$ (2)

Q.5. Find the principal solutions of $\cot\theta = 0$ (2)

Q.6. Find the value of k, if $2x + y = 0$ is one of the lines represented by $3x^2 + kxy + 2y^2 = 0$ (2)

Q.7. Find the cartesian equation of the plane passing through A(1, 2, 3) and the direction ratios of whose normal are 3, 2, 5. (2)

Q.8. Find the cartesian co-ordinates of the point whose polar co-ordinates are $\left(\frac{1}{2}, \frac{\pi}{3}\right)$. (2)

Q.9. Find the equation of tangent to the curve $y = 2x^3 - x^2 + 2$ at $\left(\frac{1}{2}, 2\right)$. (2)

Q.10. Evaluate: $\int_0^{\frac{\pi}{4}} \sec^4 x \, dx$ (2)

Q.11. Solve the differential equation $y\frac{dy}{dx} + x = 0$ (2)

Q.12. Show that function $f(x) = \tan x$ is increasing in $\left(0, \frac{\pi}{2}\right)$. (2)

Q.13. From the differential equation of all lines which makes intercept 3 on x-axis. (2)

Q.14. If $X \sim B(n, p)$ and $E(X) = 6$ and $Var(X) = 4.2$, then find n and p. (2)

SECTION – C

Attempt any EIGHT of the following questions: **[24]**

Q.15. If $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$, then find the value of x. (3)

Q.16. If angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$ is equal to the angle between the lines represented by $2x^2 - 5xy + 3y^2 = 0$, then show that $100(h^2 - ab) = (a + b)^2$. (3)

Q.17. Find the distance between the parallel lines $\frac{x}{2} = \frac{y}{-1} = \frac{z}{2}$ and $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z-1}{2}$. (3)

Q.18. If A (5, 1, p), B(1, q, p) and C(1, –2, 3) are vertices of a triangle and $G\left(r, \frac{-4}{3}, \frac{1}{3}\right)$ is its centroid, then find the values of p, q, r by vector method. (3)

Q.19. If $A(\bar{a})$ and $B(\bar{b})$ be any two points in the space and $R(\bar{r})$ be a point on the line segment AB dividing it internally in the ratio m : n then prove that $\bar{r} = \frac{m\bar{b} + n\bar{a}}{m+n}$. (3)

Q.20. Find the vector equation of the plane passing through the point A(–1, 2, –5) and parallel to the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$. (3)

Q.21. If $y = e^{m\tan^{-1}x}$, then show that $(1+x^2)\frac{d^2y}{dx^2} + (2x-m)\frac{dy}{dx} = 0$ (3)

Q.22. Evaluate: $\int \frac{dx}{2+\cos x - \sin x}$ (3)

Q.23. Solve $x + y\frac{dy}{dx} = \sec(x^2 + y^2)$ (3)

Q.24. A wire of length 36 meters is bent to form a rectangle. Find its dimensions if the area of the rectangle is maximum. (3)

Q.25. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X. (3)

Q.26. If a fair coin is tossed 10 times. Find the probability of getting at most six heads. (3)

SECTION – D

Attempt any FIVE of the following questions: **[20]**

Q.27. Without using truth table prove that
$(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \equiv p \vee q$ (4)

Q.28. Solve the following system of equations by the method of inversion
$x - y + z = 4,\ 2x + y - 3z = 0,\ x + y + z = 2$ (4)

Q.29. Using vectors prove that the altitudes of a triangle are concurrent. (4)

Q.30. Solve the L.P.P. by graphical method,
Minimize $z = 8x + 10y$
Subject to $2x + y \geq 7$,
$2x + 3y \geq 15$,
$y \geq 2, x \geq 0$ (4)

Q.31. If x = f(t) and y = g(t) are differentiable functions of t so that y is differentiable function of x and $\frac{dx}{dt} \neq 0$, then prove that:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Hence find $\frac{dy}{dx}$ if $x = \sin t$ and $y = \cos t$. (4)

Q.32. If u and v are differentiable function of x, then prove that:

$$\int uv\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\int v\,dx\right]dx$$

Hence evaluate $\int \log x\,dx$ (4)

Q.33. Find the area of region between parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (4)

Q.34. Show that: $\int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx = \frac{\pi}{8}\log 2$ (4)