BOARD QUESTION PAPER : MARCH 2019
MATHEMATICS AND STATISTICS

Note:

(1) *All questions are compulsory.*
(2) *Figures to the right indicate full marks.*
(3) *The Question paper consists of **30** questions divided into **FOUR** sections **A, B, C, D.***
- ***Section A** contains **6** questions of **1 mark** each.*
- ***Section B** contains **8** questions of **2 marks** each.(One of them has internal option)*
- ***Section C** contains **6** questions of **3 marks** each.(Two of them have internal options)*
- ***Section D** contains **10** questions of **4 marks** each.(Three of them has internal options)*

(4) *For each **MCQ,** correct answer must be written along with its **alphabet**,*
e.g., (A) ……. / (B) ……. / (C) ……. / (D) ……. etc.
*In case of **MCQs**, (Q. No. **1** to **6**) evaluation would be done for the first attempt only.*
(5) *Use of logarithmic table is allowed. Use of calculator is **not** allowed.*
(6) *In L.P.P. only rough sketch of graph is expected. Graph paper is **not** necessary.*
(7) *Start each section on new page only.*

SECTION A

Select and write the most appropriate answer from the given alternative for each question:

1. The principal solutions of $\cot x = -\sqrt{3}$ are ________. [1]

(A) $\frac{\pi}{6}, \frac{5\pi}{6}$ (B) $\frac{5\pi}{6}, \frac{7\pi}{6}$

(C) $\frac{5\pi}{6}, \frac{11\pi}{6}$ (D) $\frac{\pi}{6}, \frac{11\pi}{6}$

2. The acute angle between the two planes $x + y + 2z = 3$ and $3x - 2y + 2z = 7$ is _______. [1]

(A) $\sin^{-1}\left(\frac{5}{\sqrt{102}}\right)$ (B) $\cos^{-1}\left(\frac{5}{\sqrt{102}}\right)$

(C) $\sin^{-1}\left(\frac{15}{\sqrt{102}}\right)$ (D) $\cos^{-1}\left(\frac{15}{\sqrt{102}}\right)$

3. The direction ratios of the line which is perpendicular to the lines with direction ratios –1, 2, 2 and 0, 2, 1 are ________. [1]

(A) –2, –1, –2 (B) 2, 1, 2
(C) 2, –1, – 2 (D) –2, 1, –2

4. If $f(x) = (1+2x)^{\frac{1}{x}}$, for $x \neq 0$ is continuous at $x = 0$, then f(0) =________. [1]

(A) e (B) e^2
(C) 0 (D) 2

5. $\int \frac{dx}{9x^2+1}$ =________. [1]

(A) $\frac{1}{3}\tan^{-1}(2x)+c$ (B) $\frac{1}{3}\tan^{-1}x+c$

(C) $\frac{1}{3}\tan^{-1}(3x)+c$ (D) $\frac{1}{3}\tan^{-1}(6x)+c$

6. If $y = ae^{5x} + be^{-5x}$,then the differential equation is________. [1]

(A) $\frac{d^2y}{dx^2} = 25y$ (B) $\frac{d^2y}{dx^2} = -25y$

(C) $\frac{d^2y}{dx^2} = -5y$ (D) $\frac{d^2y}{dx^2} = 5y$

SECTION B

7. Write the truth values of the following statements: [2]

i. 2 is a rational number and $\sqrt{2}$ is an irrational number.

ii. 2 + 3 = 5 or $\sqrt{2} + \sqrt{3} = \sqrt{5}$

8. Find the volume of the parallelopiped, if the coterminus edges are given by the vectors $2\hat{i} + 5\hat{j} - 4\hat{k}$, $5\hat{i} + 7\hat{j} + 5\hat{k}$, $4\hat{i} + 5\hat{j} - 2\hat{k}$. [2]

OR

Find the value of p, if the vectors $\hat{i} - 2\hat{j} + \hat{k}$, $2\hat{i} - 5\hat{j} + p\hat{k}$ and $5\hat{i} - 9\hat{j} + 4\hat{k}$ are coplanar.

9. Show that the points A(–7, 4, –2), B(–2, 1, 0) and C (3, –2, 2) are collinear. [2]

10. Write the equation of the plane $3x + 4y - 2z = 5$ in the vector form [2]

11. If $y = x^x$, find $\frac{dy}{dx}$. [2]

12. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at (–1, –2). [2]

13. Evaluate: $\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$ [2]

14. Evaluate: $\int_0^{\frac{\pi}{2}} \sin^2 x\, dx$ [2]

SECTION C

15. In ΔABC, prove that

$\sin\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{a}\right)\cos\left(\frac{A}{2}\right)$ [3]

OR

Show that $\sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{63}{16}\right)$

16. If $A(\bar{a})$ and $B(\bar{b})$ are any two points in the space and $R(\bar{r})$ be a point on the line segment AB dividing it internally in the ratio m : n, then prove that $\bar{r} = \frac{m\bar{b} + n\bar{a}}{m+n}$ [3]

17. The equation of a line is $2x - 2 = 3y + 1 = 6z - 2$, find its direction ratios and also find the vector equation of the line. [3]

18. Discuss the continuity of the function

$$f(x) = \frac{\log(2+x)\log(2-x)}{\tan x}, \qquad \text{for } x \neq 0$$

$$= 1 \qquad \text{for } x = 0$$

at the point $x = 0$ [3]

19. The probability distribution of a random variable X, the number of defects per 10 meters of a fabric is given by

x	0	1	2	3	4
P (X = x)	0.45	0.35	0.15	0.03	0.02

Find the variance of X. [3]

OR

For the following probability density function (p.d.f.) of X, find: (i) P (X < 1), (ii) P (| X | < 1)

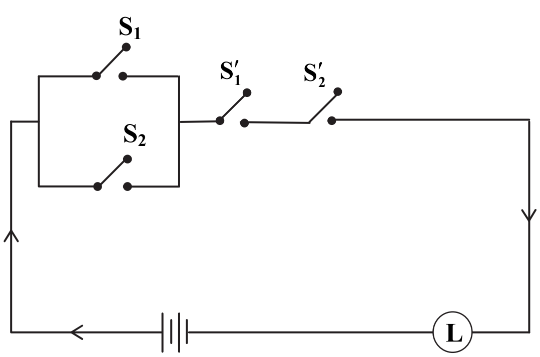
if $f(x) = \frac{x^2}{18}, \qquad -3 < x < 3$

$0, \qquad$ otherwise

20. Given is X ~ B (n, p).
If E (X) = 6, Var. (X) = 4.2, find n and p. [3]

SECTION D

21. Find the symbolic form of the given switching circuit. Construct its switching table and interpret your result.



[4]

22. If three numbers are added, their sum is 2. If two times the second number is substracted from the sum of first and third numbers we get 8 and if three times the first number is added to the sum of second and third numbers we get 4. Find the numbers using matrices. [4]

23. In ΔABC, with usual notations prove that $b^2 = c^2 + a^2 - 2ca\cos B$ [4]

OR

In ΔABC, with usual notations prove that

$$(a-b)^2\cos^2\left(\frac{C}{2}\right) + (a+b)^2\sin^2\left(\frac{C}{2}\right) = c^2.$$

24. Find 'p' and 'q' if the equation $px^2 - 8xy + 3y^2 + 14x + 2y + q = 0$ represents a pair of perpendicular lines. [4]

25. Maximize: $z = 3x + 5y$ Subject to

$x + 4y \le 24,$ $3x + y \le 21,$

$x + y \le 9,$ $x \ge 0, y \ge 0$ [4]

26. If $x = f(t)$ and $y = g(t)$ are differentiable functions of t, then prove that y is a differentiable function of x and

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \text{ where } \frac{dx}{dt} \neq 0$$

Hence find $\frac{dy}{dx}$ if $x = a\cos^2 t$ and $y = a\sin^2 t$. [4]

27. $f(x) = (x-1)(x-2)(x-3)$, $x \in [0, 4]$, find 'c' if LMVT can be applied. [4]

OR

A rod of 108 meters long is bent to form a rectangle. Find its dimensions if the area is maximum.

28. prove that : $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + c$ [4]

29. Show that: $\int_0^{\frac{\pi}{4}} \log(1 + \tan x)\, dx = \frac{\pi}{8}\log 2$ [4]

30. Solve the differential equation:

$$\frac{dy}{dx} + y \sec x = \tan x$$ [4]

OR

Solve the differential equation:

$$(x + y)\frac{dy}{dx} = 1$$