BOARD QUESTION PAPER : JULY 2019

MATHEMATICS AND STATISTICS

Time: 3 Hours **Total Marks: 80**

Note:

i. *All questions are compulsory.*
ii. *Figures to the right indicate full marks.*
iii. *The question paper consists of **30** questions divided into **FOUR** sections **A, B, C, D.***
iv. • ***Section A** contains **6** multiple choice questions (MCQ) of **1 mark** each.*
• ***Section B** contains **8** questions of **2 marks** each.(One of them has internal option)*
• ***Section C** contains **6** questions of **3 marks** each.(Two of them have internal options)*
• ***Section D** contains **10** questions of **4 marks** each.(Three of them have internal options)*
v. *For each **MCQ,** correct answer must be written along with its **alphabet**,*
e.g., (A) ……. / (B) ……. / (C) ……. / (D) ……. etc.
*In case of **MCQs**, (Q. No. **1** to **6**) evaluation would be done for the first attempt only.*
vi. Start answers of each section on new page only.
vii. *Use of logarithmic table is allowed.*
viii. *Use of calculator is **not** allowed.*
ix. *In L.P.P. only rough sketch of graph is expected. Graph paper is **not** necessary.*

SECTION A

Select and write the most appropriate answer from the given alternatives for each question: **[6]**

1. The polar co-ordinates of a point whose cartesian co-ordinates are $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ are _______. (1)

(A) $\left(1, \frac{\pi^c}{4}\right)$ (B) $\left(1, \frac{5\pi^c}{4}\right)$
(C) $\left(\sqrt{2}, \frac{\pi^c}{4}\right)$ (D) $\left(\sqrt{2}, 200^\circ\right)$

2. If a line makes angles α, β, γ with co-ordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$ _______. (1)
(A) 1 (B) – 1
(C) 2 (D) – 2

3. If the point A(λ, 5, –2) lies on the line $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$, then the value of λ is _______. (1)
(A) – 1 (B) 1
(C) 8 (D) – 8

4. The function f(x) is continuous at the point $x = 0$, where
$f(x) = \frac{\log(1+kx)}{\sin x}$, for $x \neq 0$
$= 5$, for $x = 0$
then value of k is _______. (1)
(A) – 5 (B) 5
(C) 2 (D) – 2

5. $\int \frac{(x+3)}{(x+4)^2} \cdot e^x \, dx$ is equal to _______ (1)

(A) $\frac{1}{(x+4)^2} + c$ (B) $\frac{e^x}{(x+4)^2} + c$
(C) $\frac{e^x}{x+4} + c$ (D) $\frac{e^x}{x+3} + c$

6. The differential equation whose general solution is $y = \log x + c$ is _______ (1)

(A) $x.\frac{dy}{dx} = 1$ (B) $x.\frac{dy}{dx} + 1 = 0$

(C) $\frac{dy}{dx} + x = 0$ (D) $\frac{1}{x}.\frac{dy}{dx} = 0$

SECTION B [16]

7. Write the truth values of the following statements:
   i. Two is the only even prime number.
   ii. $\cos(2\theta) = \cos^2\theta - \sin^2\theta$, for all $\theta \in R$. (2)

8. Find direction ratios of the line which is perpendicular to the lines with direction ratios 1, 3, 2 and –1, 1, 2. (2)

9. If the vectors $2\hat{i} - q\hat{j} + 3\hat{k}$ and $4\hat{i} - 5\hat{j} + 6\hat{k}$ are collinear, then find the value of q. (2)

10. Find the vector equation of the line passing through points A(3, 4, –7) and B (6, –1,1). (2)

11. Differentiate $\log(1 + x^2)$ with respect to $\tan^{-1}x$. (2)

12. The displacement 's' of a particle at time 't' is given by $s = t^3 - 4t^2 - 5t$. Find its velocity and acceleration at time t = 2 seconds. (2)

13 Solve: $\int x^x.(1 + \log x)dx$ (2)

OR

Solve: $\int \frac{10 \cdot x^9 + 10^x \log 10}{10^x + x^{10}}\, dx$

14. Find the area of region bounded by $y^2 = 24x$ and line $x = 1$. (2)

SECTION C [18]

15. In any triangle ABC with the usual notations prove that $a^2 = b^2 + c^2 - 2bc\cos A$. (3)

16. If $\bar{a}$ and $\bar{b}$ are two non zero and non collinear vectors, then prove that any vector $\bar{r}$ coplanar with $\bar{a}$ and $\bar{b}$ can be uniquely expressed as linear combination of $\bar{a}$ and $\bar{b}$. (3)

17. The co-ordinates of the foot of a perpendicular drawn from the origin to the plane are (2, 3, 1). Find the equation of the plane in vector form. (3)

OR

Find the value of μ, if the points with position vectors $\hat{i} - \hat{j} + 3\hat{k}$ and $3\hat{i} + 4\hat{j} + \mu\hat{k}$ are equidistant from the plane $\bar{r} \cdot (5\hat{i} + 2\hat{j} - 7\hat{k}) + 8 = 0$.

18. If the function $f(x)$ is continuous on its domain $[-2, 2]$ where

$f(x) = \frac{\sin ax}{x} + 2$, for $-2 \le x < 0$

$= 3x + 5$, for $0 \le x \le 1$

$= \sqrt{x^2 + 8} - b$, for $1 < x \le 2$

find the values of a and b. (3)

19. The p.d.f. of continuous random variable X is given by

$f(x) = \frac{x}{8}$, $0 < x < 4$

$= 0$, otherwise

Find i. $P(X \le 2)$
ii. $P(2 < X \le 3)$
iii. $P(X > 3)$ (3)

20. Suppose that 80% of all families own a television set. If 10 families are interviewed at random, find the probability that seven families own a television set. (3)

OR

The probability that a person undergoes a kidney operation will recover is 0.7. Find the probability that of six patients who undergo similar operations half of them will recover.

SECTION D [40]

21. Using truth table, prove that:
$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$ (4)

22. Find the inverse of matrix $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}$ by using elementary row transformations. (4)

OR

If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$, find AB and $(AB)^{-1}$.
Verify that $(AB)^{-1} = B^{-1}A^{-1}$.

23. Show that $\sin^{-1}\left(\frac{5}{13}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{63}{16}\right)$. (4)

24. Show that the equation $x^2 - 6xy + 5y^2 + 10x - 14y + 9 = 0$ represents a pair of lines. Find the acute angle between them. Also find the point of intersection of the lines. (4)

OR

Find the joint equation of a pair of lines passing through the origin each of which making an angle of 30° with the line $3x + 2y - 11 = 0$.

25. Solve the following L.P.P. using graphical method:
Minimize: $z = 8x + 10y$
Subject to $2x + y \geq 7$
$2x + 3y \geq 15$
$y \geq 2$
$x \geq 0, y \geq 0$ (4)

26. If $y = f(x)$ is a differentiable function of x such that inverse function $x = f^{-1}(y)$ exists, them prove that x is a differentiable function of y and $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$, where $\frac{dy}{dx} \neq 0$

Hence find $\frac{d}{dx}\left[\sin^{-1} x\right]$. (4)

27. The surface area of a spherical balloon is increasing at the rate of 2 cm^2/sec. At what rate is the volume of the balloon is increasing when the radius of the balloon is 6 cm. (4)

28. Solve: $\int \frac{2\sin x + 3\cos x}{3\sin x + 4\cos x} dx$ (4)

OR

Solve: $\int \sec^3(2x) dx$

29. Prove that:

$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$, if f(x) is even function

$= 0,$, if f(x) is odd function (4)

30. Solve the differential equation:
$x.y\frac{dy}{dx} = x^2 + 2y^2, y(1) = 0$ (4)